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The effect of non-condensable gas on forced convection condensation along a horizontal plate in a porous medium

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Abstract—This paper is concerned with forced convection condensation from a vapour–gas mixture on a horizontal plate embedded in a porous medium. It is found that about 50% reduction in heat transfer may be induced by the presence of only 5% non-condensable gas. Furthermore, the reduction is accentuated at lower operating pressures. By comparison it is indicated that forced convection condensation in porous media is much more sensitive to non-condensable gas than that in an open space.

1. INTRODUCTION

THE RECENT years have witnessed a growing interest in film condensation heat transfer in a porous medium [1–5]. These investigations were all concerned with film condensation in a gravity flow, i.e. in natural convection boundary layer flow.

The present paper is primarily concerned with condensation in a forced convection boundary layer flow along a horizontal plate embedded in a porous medium. This problem is presented in order to simulate the condensation process on cold base rocks when steam-driven enhanced oil-recovery techniques are used in petroleum reservoirs, and also has applications in horizontal-oriented packed-bed heat exchangers designed for sensible or latent heat, and in geothermal recovery systems. In the case of gravity-flow condensation in a porous medium, it is known that the presence of a small amount of non-condensable gas results in a remarkable reduction in condensation heat transfer. Because of this, and because non-condensable gases such as carbon and sulphur dioxide are more likely to exist in most geothermal and petroleum reservoirs, the present study will explore the forced convection condensation of vapour–gas mixtures along a horizontal plate, and aims to find the possible effect of non-condensable gas on heat transfer in such a system.

The analytical treatment is first carried out for any arbitrary combination of a vapour and a non-condensable gas, and is then applied to a steam–air mixture.

2. ANALYSIS

The situation under study is shown schematically in Fig. 1. The free stream flow is a mixture of a vapour and a non-condensable gas. The cooled horizontal plate is at a uniform temperature T_w. The free stream temperature T_∞ is the saturation temperature corresponding to the partial pressure of the vapour in the free stream. The concentration of non-condensable gas in the free stream is characterized by its mass fraction $W_{g\infty}$. The analysis is based on the following assumptions:

(1) A distinct two-phase boundary exists between the mixture and condensate film.

(2) The interface $y = \delta$ is smooth and stable, and is at an unknown temperature T_i and has an unknown concentration W_{gi}.

(3) Boundary layer approximations and Darcy's law are applicable to both phases.

(4) Properties of fluids and the porous medium are constant.

(5) The condensate film is so thin that energy convection terms may be neglected.

2.1. *Liquid boundary layer*

The liquid layer is now treated by the Nusselt model according to assumption (5), then the momentum and energy equations are

$$u = -\frac{K}{\mu_L}\frac{\partial p}{\partial x}, \quad \frac{\partial^2 T}{\partial y^2} = 0. \tag{1}$$

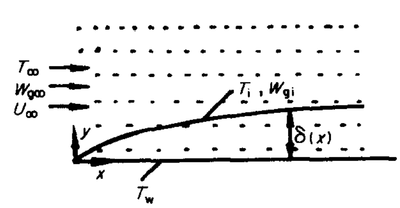

FIG. 1. Schematic of the physical model.

NOMENCLATURE

c_p	specific heat
D	binary diffusion coefficient
f	streamfunction profile
h_{fg}	latent heat
j	diffusive mass flux
k	equivalent thermal conductivity
K	permeability of porous medium
M	molecular weight
m	condensate rate
p	total pressure
p_v	vapour pressure
Pe	Peclet number, $U_\infty x/D$
Pr	Prandtl number
q	local heat flux
R	density ratio, ρ_L/ρ
Sc	Schmidt number
T	temperature
U_∞	free stream velocity
u, v	velocity components
W	mass fraction
x, y	coordinates.

Greek symbols

δ	condensate layer thickness
η	similarity variable
μ	absolute viscosity
ν	kinematic viscosity
ρ	density
Φ	dimensionless mass fraction, equation (13)
ψ	streamfunction.

Subscripts

g	non-condensate gas
i	interface
L	liquid
v	vapour
w	wall
0	condensation of pure vapours
∞	bulk.

For the free stream, Darcy's law gives

$$U_\infty = -\frac{K}{\mu}\frac{\partial p}{\partial x} \quad (2)$$

where fluid properties without a subscript correspond to average mixture properties. Eliminating $\partial p/\partial x$ between equations (1) and (2), we have

$$u = \frac{\mu}{\mu_L} U_\infty. \quad (3)$$

The solution of the temperature distribution, subject to the boundary condition that $T = T_w$ at $y = 0$ and $T = T_i$ at $y = \delta$, can be carried out easily. The results which are desired in the forthcoming sections are briefly outlined below.

(1) Local rate of heat transfer per unit area q

$$\frac{qx}{h_{fg}\mu_L} = \left[\frac{c_{pL}(T_i - T_w)}{h_{fg}\, Pr_L}\right]^{1/2}\left(\frac{\mu}{\mu_L}\right)^{1/2}\left(\frac{U_\infty x}{2\nu_L}\right)^{1/2}. \quad (4)$$

(2) Thickness of condensate film δ

$$\frac{\delta}{x} = \left[\frac{c_{pL}(T_i - T_w)}{h_{fg}\, Pr_L}\right]^{1/2}\left(\frac{\mu}{\mu_L}\right)^{1/2}\left(\frac{U_\infty x}{2\nu_L}\right)^{-1/2}. \quad (5)$$

(3) Local rate of condensation $\dot{m}$

$$\dot{m} = \frac{q}{h_{fg}}. \quad (6)$$

The foregoing results indicate that the heat transfer and condensate mass flux would be fully determined provided the interfacial temperature T_i were known. However, the value of T_i is unknown a priori in the presence of non-condensable gas, but rather, it depends on the interrelation between the transport process in the vapour–gas boundary layer and that in the liquid film.

2.2. *Vapour–gas boundary layer*

Let W_g denote the local mass fraction of the condensable gas such that

$$W_g = \frac{\rho_g}{\rho_g + \rho_v}. \quad (7)$$

Mass conservation of the mixture, species conservation for the non-condensable gas and momentum conservation consist of the governing equations for the present problem, and can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$u\frac{\partial W_g}{\partial x} + v\frac{\partial W_g}{\partial y} = D\frac{\partial^2 W_g}{\partial y^2} \quad (9)$$

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial x} \quad (10a)$$

or

$$u = U_\infty. \quad (10b)$$

By making use of the following similarity variables:

$$\eta = \frac{\sqrt{(Pe_x)}(y-\delta)}{x} \quad (11)$$

$$\psi = D\sqrt{(Pe_x)} f(\eta) \quad (12)$$

$$\Phi(\eta) = \frac{W_{\mathrm{g}} - W_{\mathrm{g}\infty}}{W_{\mathrm{gi}} - W_{\mathrm{g}\infty}} \tag{13}$$

where

$$Pe_x = \frac{U_\infty x}{D} \tag{14}$$

the governing equations (8)–(10) can be reduced to

$$f' = 1 \tag{15}$$

$$\Phi'' + \tfrac{1}{2} f \Phi' = 0. \tag{16}$$

The boundary conditions on Φ follow directly from equation (13), thus

$$\Phi(0) = 1, \quad \Phi(\infty) = 0. \tag{17}$$

An analytical solution of equations (15)–(17) can be written in the form

$$\Phi = 1 - \frac{\operatorname{erf}[(\eta + f(0))/2] - \operatorname{erf}[f(0)/2]}{1 - \operatorname{erf}[f(0)/2]} \tag{18}$$

where $f(0)$ has a value larger than zero because a suction of mass is present at the interface where condensation of vapours occurs. The value of $f(0)$ may be determined from the continuity of mass flux across the interface, which is stated as

$$\dot{m} = \rho \left(u \frac{\mathrm{d}\delta}{\mathrm{d}x} - v \right)\bigg|_{y=\delta} = \tfrac{1}{2} \rho D f(0) \frac{\sqrt{(Pe_x)}}{x}. \tag{19}$$

Substituting equation (6) into equation (19), we have

$$f(0) = \sqrt{(2Sc)} \left[\frac{Rc_{p\mathrm{L}}(T_{\mathrm{i}} - T_{\mathrm{w}})}{h_{\mathrm{fg}} Pr_{\mathrm{L}}} \right]^{1/2} \tag{20}$$

where R is the density ratio (ρ_{L}/ρ).

Of particular relevance for the present analysis is the derivative $\Phi(\eta)$ at $\eta = 0$, which follows from equation (18) as

$$\Phi'(0) = -\frac{1}{\sqrt{\pi}} \frac{\exp[-\frac{1}{4} f^2(0)]}{1 - \operatorname{erf}[f(0)/2]}. \tag{21}$$

It remains to extract the value of the interfacial concentration W_{gi} which, in turn, leads to the interface temperature T_{i}. Owing to the fact that the interface is impermeable to the non-condensable gas, it follows that the interfacial mass flux of the non-condensable gas is zero. Taking into account both convective and diffusive transport, the impermeability condition becomes

$$\rho_{\mathrm{g}} \left(u \frac{\mathrm{d}\delta}{\mathrm{d}x} - v \right) = j_{\mathrm{g}} = -\rho D \frac{\partial W_{\mathrm{g}}}{\partial y}. \tag{22}$$

Recasting equation (22) into the similarity variables of the analysis, one arrives at

$$\frac{W_{\mathrm{g}\infty}}{W_{\mathrm{gi}}} = 1 + \frac{f(0)}{2\Phi'(0)}. \tag{23}$$

It is interesting to note that $W_{\mathrm{g}\infty}$ and W_{gi} appear only as a ratio form $W_{\mathrm{g}\infty}/W_{\mathrm{gi}}$.

Table 1. Results from the velocity and diffusion solutions

$f(0)$	$Sc\,Rc_{p\mathrm{L}}(T_{\mathrm{i}} - T_{\mathrm{w}})/(h_{\mathrm{fg}} Pr_{\mathrm{L}})$	$W_{\mathrm{g}\infty}/W_{\mathrm{gi}}$
0.04	0.0008	0.9653
0.08	0.0032	0.9322
0.16	0.0128	0.8702
0.2	0.0200	0.8411
0.4	0.0800	0.7133
0.6	0.1800	0.6096
0.8	0.3200	0.5241
1.0	0.5000	0.4549
1.5	1.1250	0.3257
2.0	2.0000	0.2436
3.0	4.5000	0.1424
3.6	6.4800	0.1039
4.0	8.0000	0.0852
6.0	18.000	0.0481
8.0	32.000	0.0287
10.0	50.000	0.0189
12.0	72.000	0.0134
15.0	112.50	0.0087
20.0	200.00	0.0049

Now, attention may be turned to the determination of W_{gi}. First, if $f(0)$ is specified, $\Phi'(0)$ can be obtained from equation (21). With $f(0)$ and $\Phi'(0)$ thus available, equation (23) yields $W_{\mathrm{g}\infty}/W_{\mathrm{gi}}$. By successively prescribing $f(0)$, a corresponding set of values for $W_{\mathrm{g}\infty}/W_{\mathrm{gi}}$ can be generated. On the other hand, since there is a unique relationship between $f(0)$ and $Sc\,Rc_{p\mathrm{L}}(T_{\mathrm{i}} - T_{\mathrm{w}})/(h_{\mathrm{fg}} Pr_{\mathrm{L}})$, which is provided by equation (20), it follows that $W_{\mathrm{g}\infty}/W_{\mathrm{gi}}$ is solely dependent on the latter parameter. Such a dependence is shown in Table 1 and also illustrated in Fig. 2. It is interesting to observe that the results appearing in Fig. 2 have a remarkably universal character. The dependence of $W_{\mathrm{g}\infty}/W_{\mathrm{gi}}$ on Sc and $Rc_{p\mathrm{L}}(T_{\mathrm{i}} - T_{\mathrm{w}})/(h_{\mathrm{fg}} Pr_{\mathrm{L}})$ appears only as a product of these parameters, rather than separate dependencies, as in the case of forced convection condensation in open spaces [6].

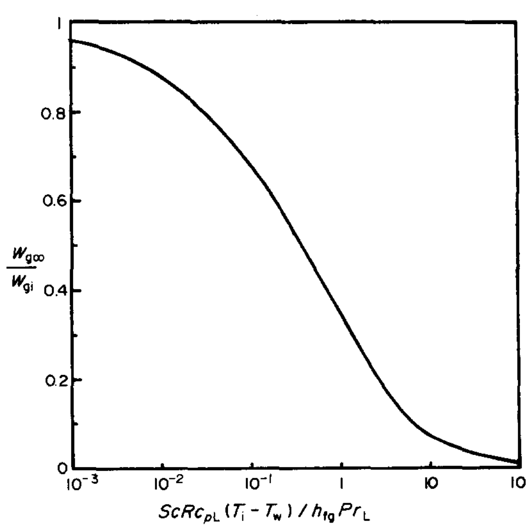

FIG. 2. Variation of $W_{\mathrm{gi}}/W_{\mathrm{g}\infty}$ with $Sc\,Rc_{p\mathrm{L}}\Delta T/(h_{\mathrm{fg}} Pr_{\mathrm{L}})$.

2.3. *Interfacial temperature and heat transfer*

Suppose that the problem has the three following quantities specified: T_∞, $W_{g\infty}$ and T_w. The first step is to evaluate the total pressure p of the system. To this end, we assume that the vapour–gas mixture and its components behave like perfect gases such that

$$\frac{p_v}{p} = \frac{1-W_{g\infty}}{1-(1-M_v/M_g)W_{g\infty}} \quad (24)$$

where p_v stands for the vapour pressure. If T_∞ is prescribed and free stream is assumed to be at saturation, then p_v is available from the tabulated properties of the vapour. With this and with the known value of $W_{g\infty}$, the total pressure p follows from equation (24).

Next, a trial value of T_i is assumed, and with this, one would compute $Sc\,Rc_{pL}(T_i-T_w)/(h_{fg}Pr_L)$. Then one can immediately read W_{gi} from the ordinate of Fig. 2. Owing to the fact that the interface is a saturation state for the vapour. Thus p_{vi} can be found from the relation

$$\frac{p_{vi}}{p} = \frac{1-W_{gi}}{1-(1-M_v/M_g)W_{gi}} \quad (25)$$

which is simply an application of equation (24) to the interface. Then, for this p_{vi}, another interfacial temperature can be obtained from the vapour tables. If this T_i differs from the initial guess for T_i, a readjustment is made and the procedure is repeated until convergence.

The procedure outlined above is easily realized on a computer. Indeed, the forthcoming numerical results of this investigation were generated in this manner.

Once T_i has been found, then the heat transfer rate follows directly from equation (4). It is particularly interesting to compare the heat transfer rate in the presence of non-condensable gas with and without non-condensable gas. The comparison is made under the condition that T_∞, T_w and U_∞ are the same in the two cases. For pure vapours, the local heat flux q_0 is obtained also from equation (4) only by substituting T_∞ for T_i. Thus the ratio q/q_0, which characterizes the influence of non-condensable gas on condensation heat transfer, consists of two terms, One is the ratio of the temperature difference $(T_i-T_w)/(T_\infty-T_w)$, and the other is the property ratio. Because the present analysis assumes constant properties, then the reference temperatures needed in evaluating properties of fluids are here chosen as

$$T^* = \tfrac{1}{2}(T_\infty+T_i) \quad (26)$$

for the vapour–gas mixture, and

$$T^* = T_w+\tfrac{1}{3}(T_i-T_w) \quad (27)$$

for the condensate liquid. In the case of pure vapours, T_i should be equal to T_∞. All thermodynamic and transport properties needed in the theory developed above are computed in the same way as in refs. [6, 7],

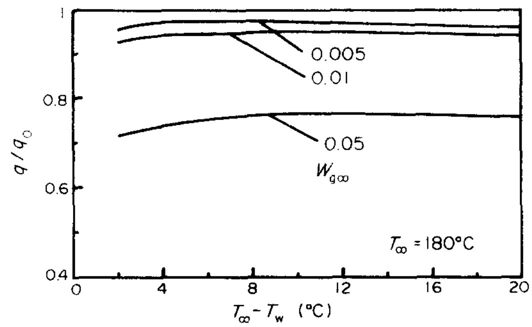

FIG. 3. The effect of non-condensable gas on forced convection condensation in porous media.

except that here k represents the equivalent thermal conductivity of the porous medium saturated with the fluid.

To sum up, it is clear that the heat transfer ratio q/q_0 can be calculated for a given free stream flow of vapour and non-condensable gas, if the values of T_∞, T_w and $W_{g\infty}$ are prescribed. The departure of q/q_0 from unity is a direct measure of the effect of the non-condensable gas.

3. CONDENSATION OF STEAM WITH AIR AS NON-CONDENSABLE GAS

The steam–air system, besides being of engineering importance, offers the possibility of direct comparison between forced boundary layer flow in a porous medium and that in an open space. For the steam–air system, we have the following parameters specified:

$$M_g = 28.97, \quad M_v = 18.02 \quad \text{and} \quad Sc = 0.55.$$

The heat transfer reductions due to non-condensable gas are presented in Figs. 3–6, which correspond respectively to T_∞ values of 180–70°C. In

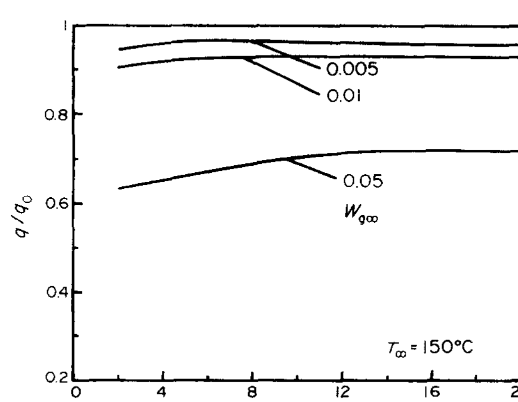

FIG. 4. The effect of non-condensable gas on forced convection condensation in porous media.

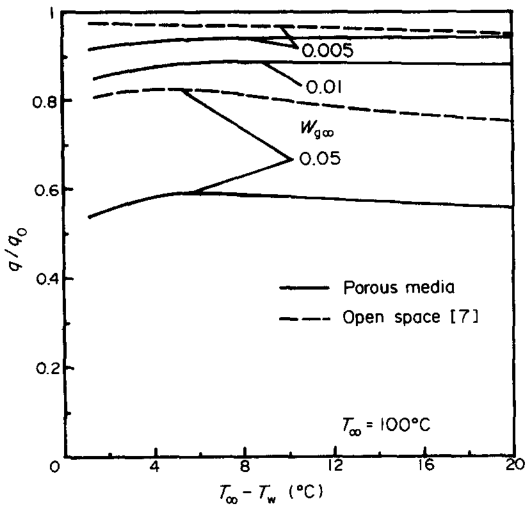

FIG. 5. The effect of non-condensable gas on forced convection condensation in porous media and in open spaces.

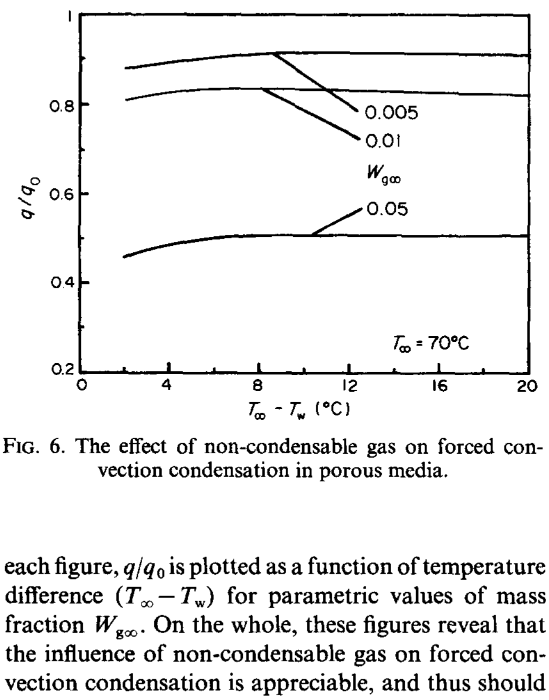

FIG. 6. The effect of non-condensable gas on forced convection condensation in porous media.

each figure, q/q_0 is plotted as a function of temperature difference $(T_\infty - T_w)$ for parametric values of mass fraction $W_{g\infty}$. On the whole, these figures reveal that the influence of non-condensable gas on forced convection condensation is appreciable, and thus should be taken into account in engineering practices. For example, 50% reductions may be sustained for bulk mass fractions $W_{g\infty}$ as small as 5%. With increasing values of $W_{g\infty}$, the heat transfer flux decreases monotonically. At a fixed value of $W_{g\infty}$, the reduction in heat transfer is very insensitive to the bulk-to-wall temperature difference, except at small temperature differences. Furthermore, by making comparisons from graph to graph, it is seen that the effect of the non-condensable gas is strongly accentuated as the bulk saturation temperature (i.e. system pressure) decreases, a trend similar to that found in refs. [6, 7].

It is particularly interesting to compare the influence of non-condensable gas on forced convection condensation in a porous medium with that in an open space. Such a comparison is plotted in Fig. 5 with dashed lines representing the results for the latter case available in ref. [6]. It is observed that there are large differences in the extent of heat transfer reductions brought about by the presence of non-condensable gas for the two cases. Forced convection condensation in a porous medium is much more sensitive to a non-condensable gas than is forced convection condensation in an open space. This may be attributed to the large diffusion resistance existing in the former case.

4. AN ENGINEERING APPLICATION

The theory and results presented above can be easily applied to engineering practices. Let us consider a typical steamflooding operation in a linear oil reservoir with a height of 3.5 m. The injection rate of steam is about 1.8×10^5 kg day^{-1}, or in other words, U_∞ which is a main parameter in this study, has the value of 0.085 m s^{-1}. The saturated steam temperature is 150°C and that of the surrounding rocks is 130°C. Some calculated results for this problem have been listed in Table 2. It is noted that about 7–14% of steam is consumed before the steam arrives at the condensation front, where oil is heated by release of latent heat of steam, due to heat losses to the surrounding rocks. Though increasing mass fractions of non-condensable gas can reduce these losses, this method is not recommended for enhancing the oil

Table 2. Results for an application example

Mass fraction of air (%)	Length of reservoir (m)	Heat loss (per width) (kW m^{-1})	Condensate rate (per width) kg (m s^{-1})$^{-1}$	Fraction of steam condensed (%)	Condensate film thickness (m)
	50	124.5	0.058	7.65	0.011
0	100	176.0	0.082	10.82	0.016
	150	215.6	0.101	13.32	0.019
	50	114.5	0.053	7.04	0.010
1	100	161.9	0.075	9.95	0.014
	150	198.4	0.093	12.25	0.017

recovery and energy efficiency, because the presence of non-condensable gas has also a detrimental effect on the release of latent heat at the condensation front.

It is seen from the table that the condensate film thickness may range from 0.01 to 0.019 m, depending on the distance from the inlet to the condensation front. It is only 1% of the half width of the reservoir. This thin condensate film and the small fraction of steam condensed along the duct ensure that the present theory for boundary layer forced flow condensation along a horizontal plate is applicable to a linear reservoir (i.e. a flat plate duct).

5. CONCLUDING REMARKS

The effect of non-condensable gas on forced convection condensation along a horizontal plate embedded in a porous medium has been investigated in this paper. It was found that the influence of the non-condensable gas is so appreciable that nearly 50% reduction in condensation heat transfer may be induced with the presence of only 5% non-condensable gas. Furthermore, these reductions in heat transfer would be accentuated at low operating pressures. From the present study, it has also been indicated that forced convection condensation in a porous medium is much more sensitive to a non-condensable gas than that in an open space. An application of the theory to a typical steamflooding oil-recovery reservoir has been illustrated.

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EFFET DES GAZ INCONDENSABLES SUR LA CONDENSATION, EN CONVECTION FORCEE, SUR UNE PLAQUE HORIZONTALE DANS UN MILIEU POREUX

Résumé—On considère la condensation en convection forcée pour un mélange vapeur–gaz sur une plaque horizontale noyée dans un milieu poreux. On trouve qu'une réduction de 50% environ du transfert de chaleur peut être induite par la présence de 5% seulement de gaz incondensable. Cette réduction est accentuée par des pressions opératoires faibles. Par comparaison, on voit que la condensation avec convection forcée dans les milieux poreux est beaucoup plus sensible aux gaz incondensables que dans un espace ouvert.

DER EINFLUSS EINES NICHTKONDENSIERBAREN GASES AUF DIE KONDENSATION BEI ERZWUNGENER KONVEKTION ENTLANG EINER HORIZONTALEN PLATTE IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Diese Arbeit beschäftigt sich mit der Kondensation eines Dampf–Gas-Gemisches bei erzwungener Konvektion an einer horizontalen, in einem porösen Medium eingebetteten Platte. Es zeigt sich, daß der Wärmeübergang durch die Anwesenheit von nur 5% nichtkondensierbaren Gases um ungefähr 50% verschlechtert wird. Diese Verringerung des Wärmeübergangs wird bei niedrigen Systemdrücken noch verstärkt. Aus dem Vergleich mit der Kondensation bei erzwungener Konvektion in einem freien Raum folgt, daß die Kondensation in einem porösen Medium wesentlich empfindlicher auf nichtkondensierbare Gase reagiert.

ВЛИЯНИЕ НЕКОНДЕНСИРУЕМОГО ГАЗА НА КОНДЕНСАЦИЮ У ГОРИЗОНТАЛЬНОЙ ПЛАСТИНЫ В ПОРИСТОЙ СРЕДЕ ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация—Исследуется конденсация парогазовой смеси на помещенной в пористую среду горизонтальной пластине при вынужденной конвекции. Найдено, что содержание даже 5% неконденсируемого газа может снизить теплоперенос примерно на 50%. Более того, это снижение усиливается при понижении рабочего давления. На основе сравнений показано, что в пористой среде конденсация при вынужденной конвекции более чувствительна к присутствию неконденсируемого газа, чем в открытом пространстве.